Assignment 3

3. Let $X_1, ..., X_{25}$ be a random sample from a normal distribution with mean μ_1 and standard deviation 10.

   a. What is the power of the test of H_0: $\mu_0 = 0$ versus H_A: $\mu_1 = 1.5$ at level $\alpha = 0.05$ of H_0: $\mu_0 = 0$?

   Sol'n. *The power of the test is the probability of rejecting the null hypothesis H_0 given that the alternative hypothesis H_A is true. With $\alpha = 0.05$, we will reject H_0 when our test statistic exceeds the critical value*

```
> alpha <- 0.05
> qnorm(1 - alpha, mean=0, sd=10/sqrt(25))
[1] 3.289707
```

   If H_A is true, then the mean will follow a normal distribution with mean 1.5 and standard deviation $10/\sqrt{25} = 2$. The probability that a random variable following $N(1.5, 2^2)$ is greater than the critical value $c_\alpha = 3.289707$ is

```
> pnorm(3.289707, mean=1.5, sd=2, lower.tail=FALSE)
[1] 0.1854327
```

   So the power of the test is 0.1854327.

   b. What is the power of the test of H_0:$\mu_0 = 0$ versus H_A: $\mu_1 = 3$ at level $\alpha = 0.05$?

   Sol'n. *The critical value c_α remains 3.289707 as in (a). However, the mean of the 25 numbers now follows a normal distribution with mean 3.*

```
> pnorm(3.289707, mean=3, sd=2, lower.tail=FALSE)
[1] 0.4424133
```

   Hence the power has increased to 0.4424133.

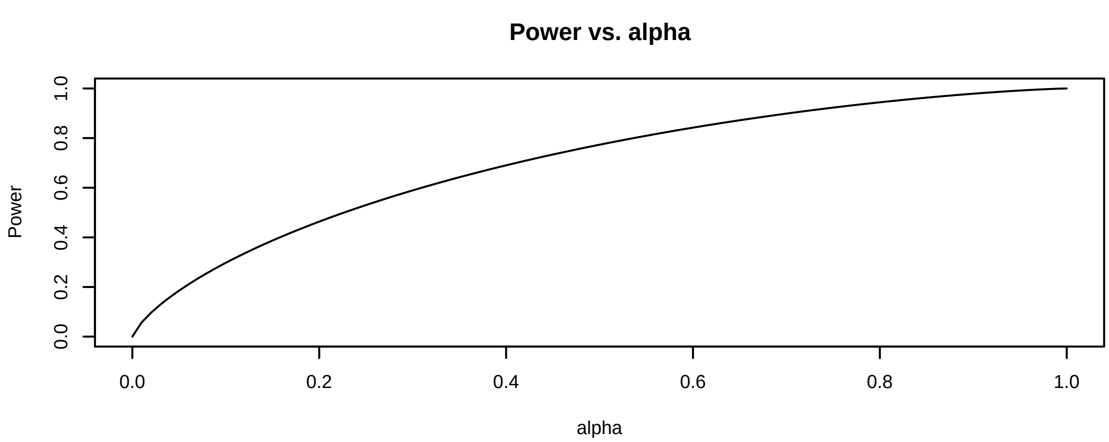
   c. Graph the power of the test of H_0 :$\mu_0 = 0$ versus H_A: $\mu_1 = 1.5$ as a function of α.

   Sol'n. *Power as a function of alpha is given by:*

```
> power <- function(alpha) 1 - pnorm(qnorm(1-alpha, mean=0, sd=2), mean=1.5, sd=2)
```

   To plot, we first create a vector of alpha values; then we apply our `power` function to each value; and then we plot the results.

```
> alpha <- seq(0, 1, by=0.01)
> plot(alpha, power(alpha), xlab="alpha", ylab="Power", main="Power vs. alpha", type="l")
```



d. How large should the sample size be, so that the test of H_0: $\mu_0 = 0$ versus H_A: $\mu_1 = 1.5$ has a power greater than 0.5 if $\alpha = 0.05$?

Sol'n. *First we modify our power function to depend on the number of samples n:*

```
> power <- function(alpha, n)
+     1 - pnorm(qnorm(1-alpha, mean=0, sd=10/sqrt(n)), mean=1.5, sd=10/sqrt(n))
```

Then we create a vector of n values and compute power for each one:

```
> ns <- 1:125
> power(0.05, ns)
  [1] 0.06747632 0.07596871 0.08301920 0.08933625 0.09519209 0.10072521
  [7] 0.10601719 0.11112073 0.11607211 0.12089750 0.12561645 0.13024402
  ...
[115] 0.48552842 0.48831099 0.49108217 0.49384195 0.49659036 0.49932740
[121] 0.50205310 0.50476746 0.50747050 0.51016223 0.51284267
```

As we can see, the first value of n such that the power is greater than 0.5 is $n = 121$.

e. How do the factors n, σ, α, $|\mu_1 - \mu_0|$ affect the power in general?

Sol'n. *Increasing any of n, α and $|\mu_1 - \mu_2|$ will increase power. Increasing σ will reduce power. The `power` function can be revised to allow each of these parameters to vary:*

```
power <- function(alpha=0.05, n=25, mu=1.5, sigma=10) {
    crit <- qnorm(1 - alpha, mean=0, sd=sigma/sqrt(n))
    1 - pnorm(crit, mean=mu, sd=sigma/sqrt(n))
}
```